Universal features in the growth dynamics of religious activities

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We quantify and analyze the growth dynamics of a religious group in 140 countries for a 47-year period (1959–2005). We find that (i) the distribution of annual logarithmic growth rates exhibits the same functional form for distinct size scales and (ii) the standard deviation of growth rates scales with size as a power law. Both findings hold for distinct measures of religious activity. These results are in surprising agreement with those found in the study of economic activities and scientific research, suggesting that religious activities are governed by universal growth mechanisms. We also compare the empirical findings on religious activities with the predictions of general models recently proposed in the context of complex organizations. Our findings should provide useful information for a better understanding of the mechanisms governing the growth of religion.

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I. INTRODUCTION

Concepts and methods from statistical physics have been applied in the study of systems in a wide variety of fields, from genetics [1] and physiology [2] to social sciences [3–10]. This approach has been shown to give useful information regarding the underlying processes responsible for the observed behavior. It has been contributing to reveal similarities between phenomena and processes in different research areas, suggesting the existence of universal mechanisms which can give rise to general laws independent of the particular details of the systems [11–14].

A striking example is found in the study of the growth dynamics of complex organizations, where similarities between economic activities [15–20] and scientific research [21–25] have been reported. Specifically, for several distinct complex organizations such as business firms, countries, universities and scientific journals, it has been found that (i) the same functional form describes the probability density function of annual growth rates, for distinct size scales and (ii) the standard deviation of growth rates decays with size as a power law. These similarities suggest that the evolution of economic and scientific activities is governed by similar growth mechanisms [16,21–24].

Here we apply a similar approach in the study of religious activities—a distinct aspect of human behavior. We explore the possibility that concepts and methods from statistical physics may lead to a better understanding of the mechanisms governing the growth of religion. Basically, we investigate (i) the distribution of annual growth rates of religious activities, for distinct size scales and (ii) the standard deviation of the growth rates in function of size. We compare our findings with previous results reported on economic activities and scientific research and find surprising similarities. We suggest possible interpretations of our findings in terms of general models recently proposed in the context of complex organizations.

In the present study, we focus on the growth dynamics of a worldwide Christian religious group—the Jehovah's Witnesses—comprising more than 6.9 million practicing members organized in about 100 000 congregations in 236 lands [26–30]. All practicing members (publishers) are actively involved in a public preaching work. Measures of their religious activities are recorded by country and published in annual service reports [26,27]. These databases have been considered a reliable—but practically unexplored—source of information about religious activities [31]. We analyze data obtained from these annual service reports which include records of 140 countries for a 47-year period (1959–2005). We focus on four distinct measures of religious activity: (i) total number of publishers; (ii) monthly average of full-time publishers (pioneers); (iii) total time (in hours) spent in the public preaching work; and (iv) monthly average of home courses conducted.

The organization of this paper is as follows. Section II presents empirical results on the growth dynamics of religious activities. Section III discusses similarities between these results and previous results on economic activities and scientific research. Section IV suggests a possible interpretation of the observed behavior in terms of general models for the growth of complex organizations. Section V contains some concluding comments.

II. DATA ANALYSIS

We consider the total number of publishers S in a given country and the annual logarithmic growth rate defined as

$$R(t) = \log_{10}\left[\frac{S(t+1)}{S(t)}\right], \tag{1}$$

where $S(t)$ and $S(t+1)$ are the total number of publishers in the successive years t and $t+1$. Figure 1 shows the temporal evolution of $R(t)$ for some countries in the period 1959–2005.

We start investigating the distribution of publishers among countries. Figure 2(a) shows the probability density of $\log_{10} S$, for 207 countries in 2005, in comparison with a Gaussian distribution. This result indicates that $\log_{10} S$ is normally distributed, implying that S is log-normally distributed. The cumulative distribution of $\log_{10} S$, shown in Fig. 2(b), reinforces this conclusion. Similar results hold for other years in the period 1959–2005.

Next, we investigate the distribution of growth rates of publishers using data for 140 countries in the period 1959–

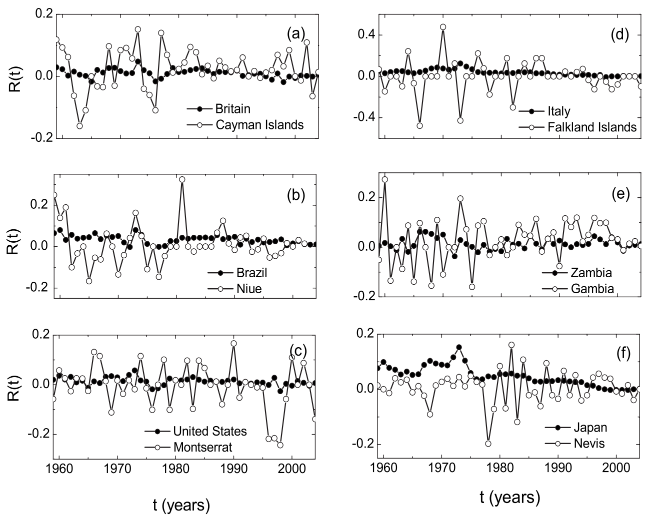

FIG. 1. Temporal evolution of growth rates $R(t)$, given by Eq. (1), for some countries in the period 1959–2005. $\bar{S}$ is the average number of publishers in the period considered. (a) Britain ($\bar{S}$=92,006) and Cayman Islands ($\bar{S}$=67), (b) Brazil ($\bar{S}$=226,682) and Niue ($\bar{S}$=20), (c) United States ($\bar{S}$=655,607) and Montserrat ($\bar{S}$=24), (d) Italy ($\bar{S}$=112,660) and Falkland Islands ($\bar{S}$=6), (e) Zambia ($\bar{S}$=66,554) and Gambia ($\bar{S}$=42), (f) Japan ($\bar{S}$=96,937) and Nevis ($\bar{S}$=40). Observe that fluctuations in R are larger in countries with small S.

2005. For most of these countries, the religious activity considered is relatively recent and, therefore, the number of publishers is a small fraction of the population of the country. At these initial stages, the growth rates are not significatively affected by geographic constraints.

In order to investigate how the distribution of growth rates depends on the number of publishers, we consider distinct groups of countries chosen according to their average number of publishers. Using data from these groups, we calculate the conditional probability density $p(R|S)$ of growth rates. Figure 3(a) shows that $p(R|S)$ is well described by a Laplace distribution

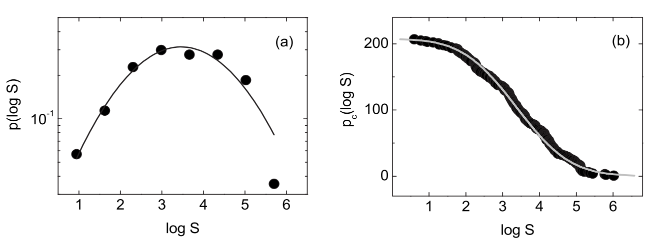

FIG. 2. Distribution of publishers. (a) Probability density of the logarithm of the number of publishers $p(\log_{10} S)$ for 207 countries in 2005. The solid line is a Gaussian fit to the data. (b) Cumulative distribution of the logarithm of the number of publishers $p_c(\log_{10} S)$ for the same data defined in (a). The solid line represents the cumulative distribution of a Gaussian variable with mean and standard deviation identical to the empirical data.

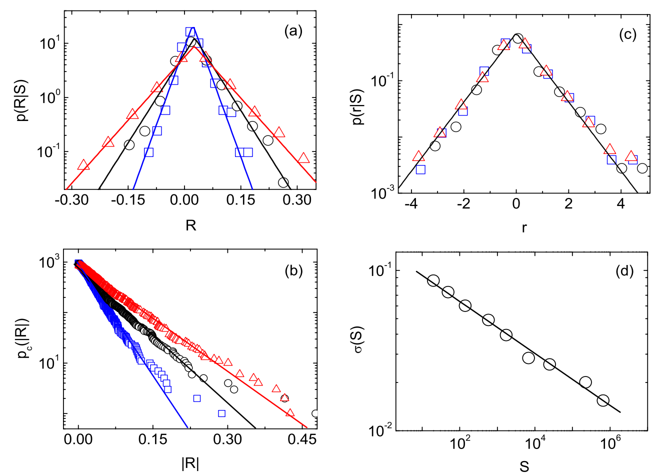

FIG. 3. (Color online) Growth dynamics of publishers. Panels (a), (b), and (c) refer to the following groups of countries selected by their average number of publishers: $6<\bar{S}<105$ (triangles), $105<\bar{S}<425$ (circles), and $980<\bar{S}<2{,}160$ (squares). We also consider the group $76{,}638<\bar{S}<655{,}607$ but, for clarity, we show data only for the three first groups. (a) Conditional probability density $p(R|S)$ of the growth rates R. The solid lines are given by Eq. (2) using μ and σ obtained directly from the data. (b) Cumulative distribution $p_c(|R|)$ of the absolute value of R. The solid lines are linear fits to the data (on monolog scale), representing exponential decay. (c) Conditional probability density $p(r|S)$ of the normalized growth rates r. All data collapse onto a single curve (solid line) given by Eq. (4). (d) Standard deviation σ of the growth rates R as a function of S. The solid line is a linear fit to the data (on log-log scale), giving a scaling exponent $\beta \simeq 0.16$.

$$p(R|S) = \frac{1}{\sqrt{2}\sigma(S)} \exp\left(-\frac{\sqrt{2}|R-\mu|}{\sigma(S)}\right), \tag{2}$$

where μ and σ are, respectively, the mean and the standard deviation of R calculated over a given group of countries. In Fig. 3(b) we show the cumulative distribution of the absolute value of R in comparison with exponential functions.

We also calculate the conditional probability density $p(r|S)$ of the normalized growth rate

$$r(t) = \frac{R(t)-\mu}{\sigma}. \tag{3}$$

Figure 3(c) shows that $p(r|S)$ appears to collapse onto a single curve,

$$p(r|S) = \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|r|). \tag{4}$$

The results shown in Figs. 3(a)–3(c) suggest that the distribution of growth rates exhibits the same functional form (Laplace distribution) for distinct size scales—a type of scale invariance.

Furthermore, we investigate how the width of the distribution of growth rates scales with size. We calculate σ as a function of S. As shown in Fig. 3(d),

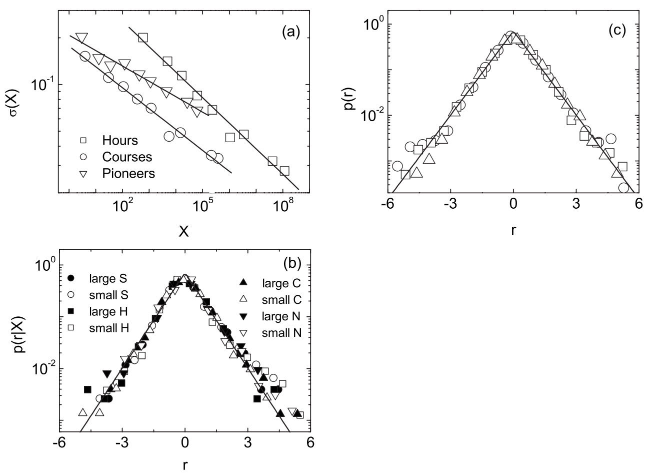

FIG. 4. Growth dynamics of other measures of religious activity. (a) Standard deviation σ of growth rates as a function of the following measures of size (X): hours H (squares), courses C (circles), and pioneers N (triangles). The solid lines are linear fits to the data (on log-log scale) giving $\beta \simeq 0.16$ for hours, $\beta \simeq 0.13$ for courses, and $\beta \simeq 0.10$ for pioneers. The curve for pioneers was vertically shifted for better visualization. (b) Conditional probability density functions $p(r|S)$, $p(r|H)$, $p(r|C)$, and $p(r|N)$. These distributions were obtained, respectively, from the following groups of countries: $979 < \bar{S} < 2157$ and $123 < \bar{S} < 424$; $230{,}508 < \bar{H} < 596{,}327$ and $26{,}431 < \bar{H} < 79{,}627$; $1{,}014 < \bar{C} < 2{,}664$ and $122 < \bar{C} < 390$; $113 < \bar{N} < 328$ and $15 < \bar{N} < 44$. The solid line is given by Eq. (4). (c) Probability density $p(r)$ of normalized growth rates r, calculated for 140 countries, for the following variables: C/S (squares), N/S (triangles), and H/S (circles). The solid line is given by Eq. (4).

$$\sigma(S) \propto S^{-\beta}, \tag{5}$$

with $\beta \simeq 0.16$, indicating that fluctuations in the growth rates decay with size as a power law. This result is consistent with those shown in Fig. 1, where fluctuations in R are larger for countries with small S.

We perform a parallel analysis for other measures of religious activities. Specifically, we consider (i) the monthly average of full-time publishers or pioneers (N), (ii) the total time (in hours) spent in the public educational work (H), and (iii) the monthly average of home courses conducted (C). Our results indicate that the analogous of Figs. 2 and 3 hold for each one of these measures. Specifically, we find that (i) the probability density of $\log_{10} N$, $\log_{10} H$, and $\log_{10} C$ are consistent with a Gaussian distribution, implying that N, H, and C are log-normally distributed, (ii) the standard deviation of growth rates scales with size as a power law, with exponents $\beta \simeq 0.16$ for hours, $\beta \simeq 0.13$ for courses and $\beta \simeq 0.10$ for pioneers [Fig. 4(a)], and (iii) the conditional probability density of normalized growth rates, $p(r|N)$, $p(r|H)$, and $p(r|C)$, are consistent with the universal curve defined in Eq. (4) [Fig. 4(b)].

Finally, we investigate the distribution of growth rates for the following variables: courses by publisher (C/S), pioneers by publisher (N/S), and hours by publisher (H/S). Figure 4(c) shows the distribution of normalized growth rates, for 140 countries in the period 1959–2005, in comparison with Eq. (4). The results shown in Fig. 4 suggest that the same general laws hold across distinct measures of religious activity.

III. UNIVERSAL FEATURES

Our findings on the growth dynamics of religious activities indicate that (i) the distribution of sizes is log-normal (Fig. 2), (ii) the distribution of annual growth rates exhibits the same functional form (Laplace distribution) for distinct size scales (Figs. 3 and 4), and (iii) the standard deviation of growth rates decays with size as a power law (Figs. 3 and 4). These findings are in remarkable agreement with results reported on the growth dynamics of economic activities and scientific research.

For example, let us consider previous results on the growth of business firms [15], countries [16] and university research [21]. The results obtained in all these cases are very similar: (i) the distribution of sizes is log-normal, (ii) the distribution of annual growth rates exhibits exponential decay for distinct size scales, and (iii) the standard deviation of growth rates decays with size as a power law. Note that the analogous of Figs. 2–4 holds to distinct measures of economic and scientific activities.

Most of the recent results on the growth of complex organizations focus on the Laplace shape of the distribution of growth rates [15–17,19,21–25]. Other results suggest that the distribution of growth rates is Laplace in the center and power law in the tails [18,20]. In any way, the same functional form, at least in the center of the distribution, describes the probability density of annual growth rates for religious, economic and scientific activities.

Concerning the size dependence of fluctuations in the growth rates, most of the previous results in this field indicate that the standard deviation σ scales with size as a power law, with exponent $\beta < 0.5$ [15–25]. In Table I, we show the power law exponent β for some selected complex organizations. Observe that β for economic activities is in quantitative agreement with our findings on religious activities, mainly for publishers and hours. This comparison suggests an additional (quantitative) similarity between the growth dynamics of religious and economic activities.

Our results are consistent with the hypothesis that the growth dynamics of complex organizations are governed by universal mechanisms, which can give rise to general laws independent of particular details of the systems. Our findings suggest the presence of universal mechanisms—similar to those found in economics and science—underlying the growth of religion. Next, we explore this possibility comparing our empirical findings with the predictions of general models for the growth of complex organizations.

IV. STOCHASTIC MODELING

Several models have been applied in the study of religion—from social diffusion mechanisms to preferential attachment processes [10,32–34]. Here, we focus on models recently proposed in the context of complex organizations in

TABLE I. Power law exponent β for some specific economic, scientific, and religious activities.

Economic activities		β
firms	sales	0.15 [15]
	employees	0.16 [15]
	assets	0.17 [15]
	cost of goods sold	0.16 [15]
	property and equipment	0.18 [15]
countries	GDP	0.15 [16]
Scientific research		β
universities	R&D	0.25 [21]
	papers	0.25 [21]
	patents	0.25 [21]
	grants	0.25 [21]
	income	0.25 [21]
scientific journals	citations	0.22 [23]
Religious activities		β
religious group [26]	publishers	0.16
	hours	0.16
	courses	0.13
	pioneers	0.10

order to gain some insight on the study of religion.

Consider, for example, a model that mimics a common characteristic of many organizations [19]—their complex internal structure—and that has been applied in the study of business firms [19], countries [16], and universities [21]. According to this model, each unit—firm, country, or university—is made up of many subunits. This statement is consistent with the fact that firms are comprised of divisions, universities are made up of schools or colleges, and so on. The model assumes that the size of each subunit evolves following a random multiplicative process and that there are interactions between subunits [19].

A similar complex internal structure is found in religious organizations, where members in each country (unit) are organized in congregations (subunits). Furthermore, the predictions of the general model proposed in Ref. [19] are in good agreement with the empirical results shown in Figs. 2–4. These facts suggest that the universal patterns observed in the growth dynamics of religious activities may arise from an interplay between complex internal structure and random multiplicative growth [19].

In contrast with the process mentioned above, the Gibrat's model [35]—the simplest model for firm growth—assumes that each organization can be viewed as a structureless unit. Gibrat's model assumes that consecutive growth rates are independent of size and uncorrelated in time. However, this process fails to explain the observed non-Gaussian distribution of growth rates and the power law dependence of the standard deviation of growth rates with size. For this reason, Gibrat's model has been modified and extended in order to give more realistic predictions [9,15,23].

Recently, we proposed a model—an extension of Gibrat's model for firm growth—which assumes that consecutive growth rates are dependent of size and correlated in time [23]. This process may offer a simple but generic approach to study the growth dynamics of complex organizations. Next, we describe the model and compare their predictions with our empirical results on religious activities.

Suppose that the total number of publishers in a given country $S(t)$ follows the rule [23]

$$S(t+1) - S(t) = \lambda(t) S^q(t), \tag{6}$$

with $q > 0$. The noise $\lambda(t)$ is given by a random multiplicative process

$$\lambda(t) = [a_0 + a_1 \lambda(t-1)]\epsilon(t), \tag{7}$$

where a_0 and a_1 are positive constants and $\epsilon(t)$ is a random number following a Gaussian distribution with zero mean and unit variance. If $q=1$, $a_1=0$, and a_0 small, Gibrat's model is recovered.

Basically, the dynamics of the model is controlled by the parameters q and a_1 [23]. If $q<1$, fluctuations are amplified for small S and weakened for large S. If $q>1$, the effect is opposed. In this way, parameter q governs the dependence of σ on S. Specifically, the model predicts $\sigma \propto S^{-\beta}$, with $\beta \simeq 1 - q$. In contrast, the exponent β is insensible to changes in a_1. The parameter a_1 reproduces a memory effect for the growth rates. It is responsible for a crossover from a Gaussian ($a_1 = 0$) to a non-Gaussian ($a_1 \neq 0$) distribution of growth rates. This crossover in the shape of the distribution of growth rates is insensible to the parameter q.

Typical realizations of this process for a particular set of parameters are shown in Figs. 5 and 6. Observe that there is a good agreement between the predictions of the model and the empirical results shown in Figs. 1, 3, and 4. This finding suggests that random multiplicative process plays an important role in the growth of complex organizations, including religious organizations.

V. CONCLUSION

In summary, by using concepts and methods from statistical physics, we have shown that a specific religious group belongs to a class of complex organizations which exhibits non-Gaussian behavior and scale invariance. Specifically, we have verified that (i) the sizes are log-normally distributed, (ii) the distribution of annual logarithmic growth rates exhibits a non-Gaussian behavior, being well described by a Laplace distribution, for distinct size scales, and (iii) the standard deviation of the growth rates scales with size as a power law.

Similar patterns are also found in the growth dynamics of economic activities and scientific research. These similarities are consistent with the principle of universality in complex systems, suggesting the presence of universal mechanisms underlying the growth dynamics of religious activities. Since universal mechanisms are independent of the particular details of the system, our findings raise the intriguing possibility of applying or extending general models for organization

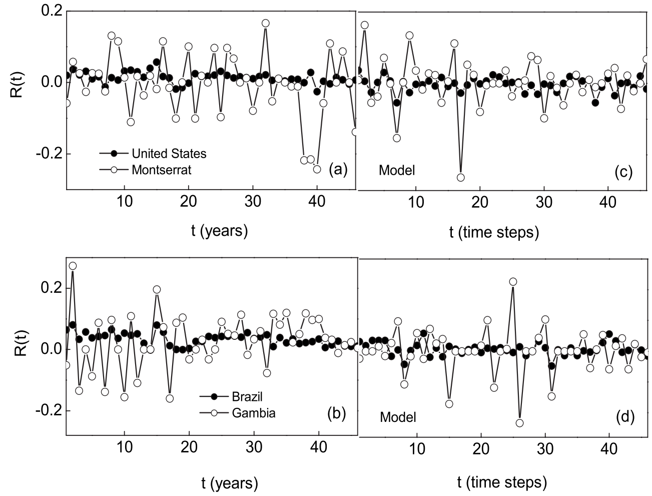

FIG. 5. Temporal evolution of growth rates: comparison between empirical data and simulations. (a) and (b) Temporal evolution of the empirical growth rates defined in Eq. (1) for selected countries in the 47-year period considered. (c) and (d) Temporal evolution of growth rates obtained by a typical realization of the model given by Eqs. (6) and (7) for 47 time steps. We use the empirical initial number of publishers as initial condition and choose a_0=0.18, a_1=0.50 and q=0.85. In order to avoid negative values of S we introduce a lower cutoff so that $S \geq 1$.

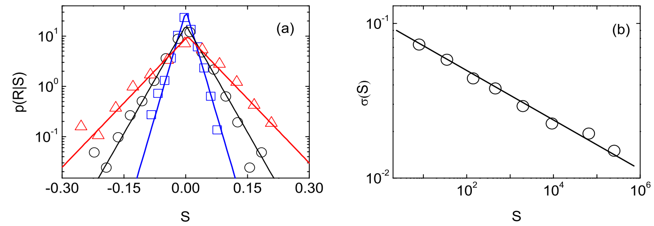

FIG. 6. (Color online) Statistical properties of growth rates: predictions of the model. Panels (a) and (b) refer to a typical realization of the model given by Eqs. (6) and (7), with a_0=0.18, a_1=0.50, q=0.85, and a cutoff $S \geq 1$. (a) Conditional probability density of growth rates $p(R|S)$ for three distinct groups of countries selected by their average value of S: small $\bar{S}$ (triangles), medium $\bar{S}$ (circles), and large $\bar{S}$ (squares). The solid lines are given by Eq. (2) using μ and σ obtained directly from the simulated data. Compare with Fig. 3(a). (b) Standard deviation σ of the growth rates R as a function of S. The solid line is a linear fit to the simulated data (on log-log scale), giving a scaling exponent $\beta \simeq 0.15$. Compare with Fig. 3(d).

growth in order to gain insight on religious phenomena. In particular, we have shown that our results on religious activities are consistent with two recently proposed models for the growth of complex organizations—both based on random multiplicative processes.

Our empirical results should be considered as targets that any empirically accurate model for the evolution of religious activities must hit, as well as a guide to develop new models. Moreover, our findings may reinforce the plausibility of applying physics methods and economic theories to the study of religious phenomena.

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